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Microstates of position and momentum result in gravitational entropy

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Abstract: Measurements of a black hole's position are limited in four different ways: Absorption of short-wavelength photons by the black hole, gravitational lensing's interference with geometric diffraction, gravitational redshift decreasing the resolution of interactions close to the event horizon, and the relatively long wavelength of Hawking radiation. These limitations mean that a black hole cannot be localized more precisely than its Schwarzschild radius. Limitations on measuring mass and velocity mean that the position and momentum of a black hole cannot be simultaneously known more precisely than $2\,h\;r_s/l_P$, a value more restrictive than the Heisenberg uncertainty principle. Hidden information about a black hole's position and momentum results in many possible microstates that are indistinguishable to an observer. One way to interpret the physical meaning of Bekenstein–Hawking entropy is as a measure of the number of these microstates. This interpretation allows entropy to be generalized to objects in any gravitational field, because gravitational redshift increases uncertainty about position and momentum for objects in all gravitational fields, not just those of black holes. © *2022 Physics Essays Publication.*
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Résumé: Les mesures de la position d'un trou noir sont finies par quatre limitations différentes: L'absorption des photons à faible longueur d'onde par le trou noir, l'interférence de la lentille gravitationnelle avec la diffraction géométrique, le décalage vers le rouge gravitationnel diminuant la résolution des interactions proches de l'horizon des événements, et la relativement grande d'onde longue du rayonnement de Hawking. Ces limitations dénotent qu'un trou noir ne peut pas être localisé plus précisément que son rayon de Schwarzschild. Les limitations sur la mesure de la masse et de la vitesse signifient que la position et le moment d'un trou noir ne peuvent pas être connus simultanément plus précisément que $2\,h\;r_s/l_P$, une valeur plus restrictive que le principe d'incertitude de Heisenberg. Les informations cachées sur la position et le moment d'un trou noir se traduisent par de nombreux micro-états possibles qui sont indiscernables pour un observateur. Une façon d'interpréter la signification physique de l'entropie de Bekenstein-Hawking consiste à mesurer le nombre de ces micro-états. Cette interprétation permet de généraliser l'entropie aux objets dans n'importe quel champ gravitationnel, vu que le décalage vers le rouge gravitationnel augmente l'incertitude sur la position et le moment des objets dans tous les champs gravitationnels, pas seulement ceux des trous noirs.

Key words: Entropy; Gravity; General Relativity; Black Hole Thermodynamics.

I. INTRODUCTION

Bekenstein and Hawking established that black holes have entropy, but there remains uncertainty about the physical meaning of this entropy.[1] One possibility is that black hole entropy represents hidden information about the position of segments of the black hole horizon.[2] Uncertainty in the position of segments of the black hole will also result in uncertainty about the momentum of the black hole, as position must be measured twice in order to calculate velocity. This paper explores the possibility of black hole entropy being related to this hidden information about position and momentum. It describes the constant relationship between the peak luminosity wavelength of Hawking radiation and Schwarzschild radius. It proposes a new uncertainty principle that describes the limits on simultaneously measuring the position and momentum of a black hole and describes how entropy can be generalized to objects in any gravitational field.

II. BLACK HOLES AND UNCERTAINTY

The Heisenberg uncertainty principle limits how much can be known simultaneously about the momentum and position of any object, including black holes. The uncertainty position can be written as[3]

$$m\,\Delta v\,\Delta x > h. \qquad (1)$$

According to this inequality, as the mass of the black hole increases, it's position and velocity can be known with greater and greater precision. For black holes, Eq. (1) is not accurate because it neglects the effects of gravity. As a gravitational field becomes stronger, there is greater uncertainty in a black hole's position and velocity. The increased

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uncertainty applies to any method of measurement, including measuring the black hole directly with a photon, indirectly by detecting a particle near the event horizon, or passively be detecting Hawking radiation.

First, consider the limitations on directly detecting a black hole with a photon. Any photon with a wavelength equal to or less than the Schwarzschild radius that directly interacts with the black hole will be absorbed. Photons with a wavelength longer than black hole's radius will diffract, but the resolution of measurements made with these photons will be larger than the Schwarzschild radius.[3]

In classical optics, the Arago spot, a bright spot directly behind a spherical object due to Fresnel diffraction, has been proposed as a precise way to measure position with long-wavelength light.[4] This technique cannot be used for precision measurements of black holes because gravitational lensing blurs the geometric scattering pattern.[5]

The black hole's location could also be determined indirectly, by measuring the position of another particle nearby. This method is also limited by the black hole's gravitational field. In empty space, the smallest length that can be measured is limited by the size of the smallest possible black hole. Distances cannot be measured to less than about one Planck length because a photon energetic enough to do this has about a Planck mass of energy, enough energy to create its own black hole.[6] Near a black hole the smallest distance that can be measured is even larger than the Planck length, because a photon with about a Planck mass of energy near the object will be gravitationally redshifted by the time it reaches a distant observer. The following equation can be used to calculate gravitational redshift:

$$\frac{\lambda_\infty}{\lambda_e} = \left(1 - \frac{r_s}{R_e}\right)^{-\frac{1}{2}}. \tag{2}$$

Using Eq. (2), a photon from an interaction one Planck length from a Planck mass black hole will be redshifted to ~1.7 times its original wavelength. This redshift leads to a loss of spatial resolution. In the above example, a ~1.7 × loss of resolution is less than the Schwarzschild radius of 2 Planck lengths, but a single interaction is not enough to precisely locate the black hole on the x-axis. Instead, it only defines where the black hole is not. For the black hole to be precisely located in this fashion would require interactions to be recorded on either side of the event horizon, which would make the uncertainty greater than the Schwarzschild radius.

Finally, one could try to locate the black hole passively, by detecting Hawking radiation. The Wein displacement law gives the wavelength of peak radiance for a black body depending on temperature. The Wein displacement is not the minimum wavelength of the thermal radiation, but given the short half-life of small black holes, and the low luminosity of large black holes, it is a reasonable approximation. The Wein displacement law is given by the following equation, where b is Wein's displacement constant (~2.987×10^{-3} m K)

$$\lambda = \frac{b}{T}. \tag{3}$$

The temperature of Hawking radiation is given as[7]

$$T = \frac{\hbar c^3}{8\pi G k_B M} \approx \frac{1.2 \times 10^{23}\, K}{M}. \tag{4}$$

Substituting Eq. (4) for the temperature in Eq. (3) gives the following equation for peak luminosity wavelength depending on black hole mass (M)

$$\lambda \approx 2.4 \times 10^{-26}\,\mathrm{m\,kg^{-1}}\, M. \tag{5}$$

The Schwarzschild radius is given by the following equation:

$$r_s = \frac{2GM}{c^2} \approx 1.5 \times 10^{-27}\,\mathrm{m\,kg^{-1}}\, M. \tag{6}$$

Since both the peak luminosity wavelength and the Schwarzschild radius are directly proportional to the black hole's mass, there is a constant relationship between peak luminosity wavelength and Schwarzschild wavelength for any size black hole, shown in Eq. (7)

$$\frac{\lambda}{r_s} \approx \frac{2.4 \times 10^{-26}\,\mathrm{m\,kg^{-1}}\, M}{1.5 \times 10^{-27}\,\mathrm{m\,kg^{-1}}\, M} \approx 16. \tag{7}$$

Equation (7) shows that for any size Schwarzschild black hole, the peak luminosity wavelength is about 16 times the Schwarzschild radius. Since position cannot be resolved to less than the wavelength of light used to do a measurement, the peak luminosity wavelength of Hawking radiation cannot be used to locate the black hole to less than about 16 times the Schwarzschild radius of the black hole.

As a result of these limitations, the position of a black hole cannot be measured to less than the Schwarzschild radius. Since calculating velocity requires two measurements of position, the limitation on measuring position will also affect uncertainty in velocity. Assuming that the measurements have to be separated by at least one unit of Planck time, the uncertainty in velocity would be twice the Schwarzschild radius (the combination of two measurements has twice the range of a single measurement) divided by the Planck time

$$\Delta v > \frac{2r_s}{t_P} = \frac{2r_S c}{l_p}. \tag{8}$$

The mass of a black hole could be measured by its diffraction of light with a wavelength longer than the Schwarzschild radius. This method would be limited in accuracy to the mass equivalent of the energy of a photon with the Schwarzschild radius, because such a photon would be absorbed by the black hole. Combining uncertainty in mass, velocity, and position gives Eq. (9), an uncertainty principle for black holes

$$m\,\Delta v\,\Delta x > \left(\frac{h}{r_S c}\right)\left(\frac{2r_S c}{l_p}\right) r_s = 2h\frac{r_s}{l_p}. \tag{9}$$

The uncertainty in Eq. (9) results in multiple possible arrangements of position and momentum for the black hole that are indistinguishable to an observer.

III. GRAVITATIONAL ENTROPY

To see how uncertainty in the black hole's position and momentum results in entropy, consider a semiclassical toy model in which the mass of a black hole is known, and its shape is assumed to be free from quantum fluctuations. In this model, a Planck mass black hole with a Schwarzschild radius of 2 Planck lengths would have two possible positions on the x-axis that are indistinguishable to an observer. These positions can be represented by 0 and 1. The three spatial axes can be defined by (x, y, z), and the number of possible positions for this black hole is given by the following set:

$$(0,0,0)\ (0,0,1)\ (0,1,0)\ (0,1,1)\ (1,0,0)\ (1,0,1)\ (1,1,0)\ (1,1,1).$$

These eight positions correspond to r^3 possible microstates of position, a relationship that holds true for any size black hole. If position is measured a second time, another set of r^3 positions is possible. Since each second position gives a unique velocity, when compared to a single possibility from the first set, the total number of microstates in this model is r6. These microstates represent an indistinguishable set of possible states that increases with the size and entropy of a black hole. Therefore, microstates of position and momentum are a candidate to explain Bekenstein–Hawking entropy.

If black hole entropy can be explained by microstates of position and momentum, then the concept of entropy can be expanded to anything within the black hole's gravitational field. Entropy applies to any object in the field, not just the black hole itself, because anything in the field will have limitations on the measurement of position and momentum due to gravitational redshift. Since gravity falls off with an inverse square, gravitational entropy can be approximated based on the black hole's entropy and the inverse square of the distance, as

$$S_G \approx S_{\mathrm{BH}}\left(\frac{r}{r_s}\right)^{-2}. \tag{10}$$

Indeed, gravitational entropy can be generalized to for objects within any gravitational fields, not just those from black holes, because all gravitational fields hide information with gravitational redshift. Extending this concept further, systems of gravitational objects, like stars, would have more gravitational entropy than the sum of the individual stars' entropy because each star's gravitational field would increase the number of microstates of position and momentum for the other stars in the system, not just for an observer.

IV. DISCUSSION AND CONCLUSIONS

This paper provides a limit on how well the position and momentum of a black hole can be measured. It also provides a rationale for how this uncertainty could result in black hole entropy, and how entropy can be generalized to any object within a gravitational field. Further work will be needed to show if the entropy predicted matches with the expected entropy from black hole thermodynamics, and to define the entropy of objects in gravitational fields.

The number of microstates predicted by the above toy model was the radius in Planck units to the sixth power. Although this represents a large number of possibilities, it does not increase as quickly the logarithmic functions typically associated with entropy. Some of this difference may be due to the simplified features of the model. Bianchi was able to reproduce the Bekenstein–Hawking expression by analyzing quantum uncertainty in the positions of individual segments of the event horizon.[2] A similar approach accounting for each segment of the black hole horizon may improve the predictions of the above model.

Both Bianchi's model and others have more closely matched Bekenstein–Hawking entropy than the toy model described above.[2,8] That said, microstates of position and momentum are worth further study because they closely match the definition of entropy used in statistical mechanics, and because they extend the concept of entropy to any object within a gravitational field. This extension of entropy may shed light on unexplained phenomena.

The entropy scale factor (ESF) is a recently published theory that proposes that entropy causes gravity, instead of the energy and momentum of general relativity.[9,10] Since the stars and black holes in a galaxy will increase microstates of position and momentum for each other as well as for an outside observer, the observer will calculate more entropy for the galaxy a whole than for the sum of the constituent gravitational bodies. In the ESF, this extra entropy will cause extra gravity, compared to what is predicted by the mass of the gravitational bodies, which may help explain phenomena attributed to dark matter.

Tests of general relativity are typically based on simple systems. Tests of binary pulsars, for example, measure how the two pulsars affect each other, not how the system of two stars would affect a third object some distance away.[11] Since these tests do not assess how the two pulsars' fields combine to affect a third object, they have not ruled out the possibility that the fields combine differently than predicted by general relativity. This means that a theory of gravity based on entropy has the possibility of making new predictions for systems while not violating tests of general relativity.

Other work on gravity's influence on the fundamental limit of measuring distance has shown how the energy of the measuring photon would warp spacetime.[12] The analysis in this paper ignores the energy of the photon to focus on the effects of the black hole itself. Future research on the interaction between both of these sources of uncertainty may further illuminate measurements of minimal distances.

Additional work is needed to explore the limitations of measuring black holes, the origin of black hole entropy, and expanding entropy to objects in gravitational fields. The framework provided above may enable further research into these areas.

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